NACRE II:
an update and extension of the NACRE* compilation of charged-particle-induced thermonuclear reaction rates for astrophysics

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*C. Angulo, M. Arnould, M. Rayet, et al., Nuclear Physics A, 656, 3-183 (1999)

Outline

1. Introduction

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   A. Experimental data selection
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   Comparison of reaction rates and S-factors (S(0))

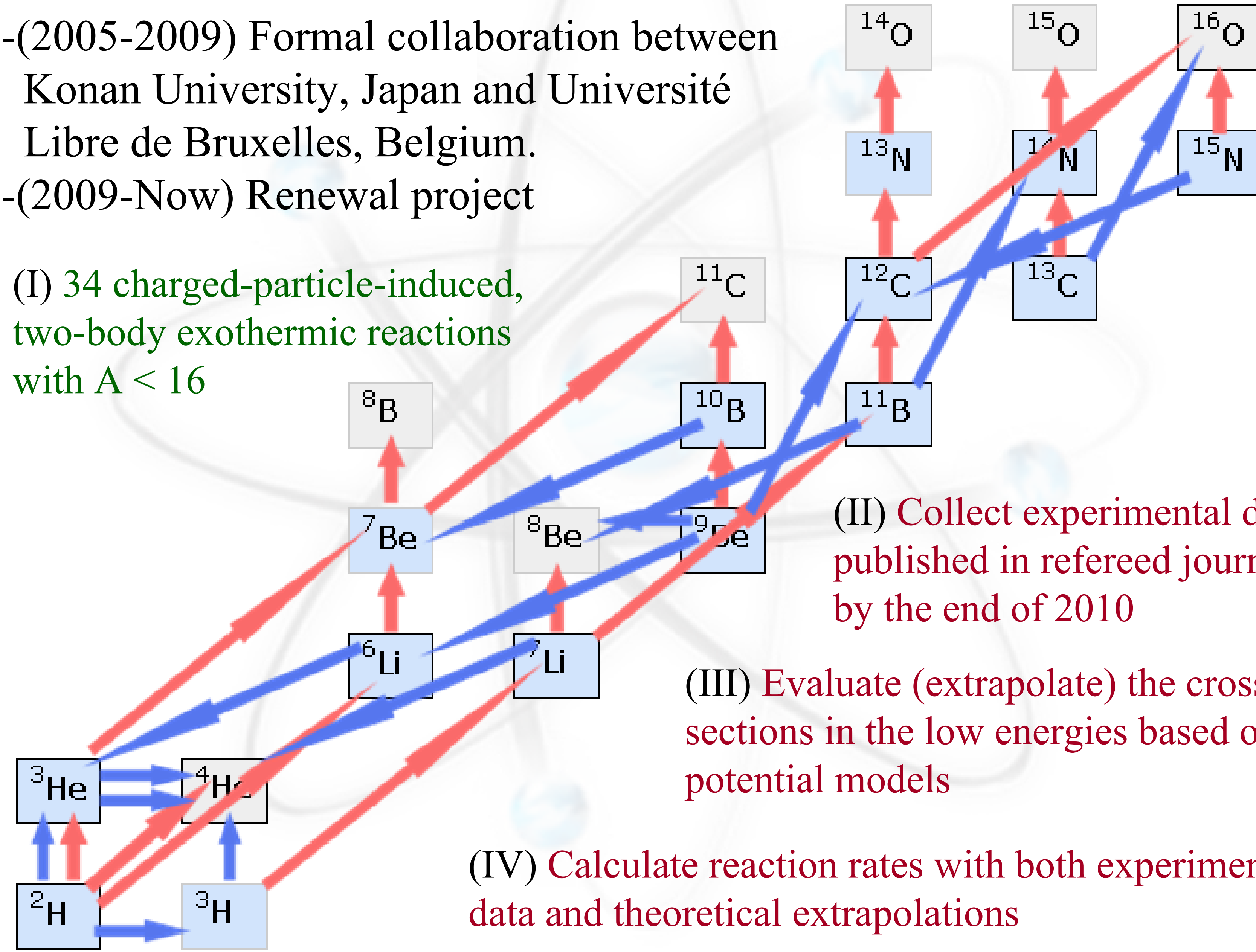
4. Summary

1. Introduction

-(2005-2009) Formal collaboration between Konan University, Japan and Université Libre de Bruxelles, Belgium.

-(2009-Now) Renewal project

(I) 34 charged-particle-induced, two-body exothermic reactions with A < 16



(II) Collect experimental data published in refereed journals by the end of 2010

(III) Evaluate (extrapolate) the cross sections in the low energies based on potential models

(IV) Calculate reaction rates with both experimental data and theoretical extrapolations

2. Theory and Methodology

A. Experimental data selection

Data Source:

NACRE II includes the experimental low energy cross sections:

(1) Collected in NACRE (before 1999)

(2) Published in referred journals (1999-2010)

Data type:

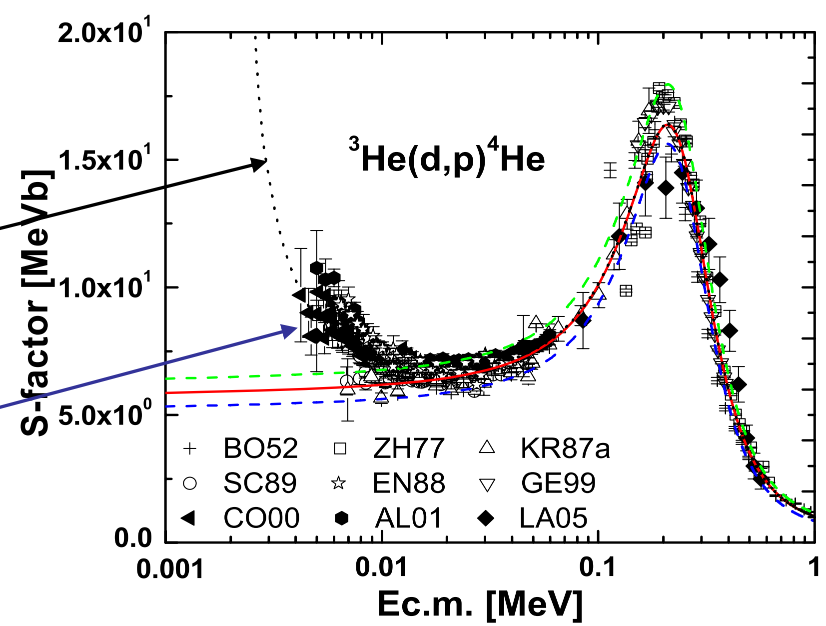
-Integral cross sections with associated errors are adopted.

-Differential quantities even measured are not considered.

"Laboratory screening"

Results by multiplying the enhancement factor (exp(πηUe/E)) to adopted extrapolation S-factors

Enhanced experimental data are disregarded.



B. Nuclear reaction model

Phenomenological method
(R-matrix)

First principle
(microscopic model, "*ab initio*")

The general potential models are adopted. Zero range distorted wave Born approximation (ZRDWBA) for transfer reactions, Potential Model (PM) for radiative capture reactions.

$$\sigma_{J_B^\pi}(E) = C_{\alpha,\beta}\frac{1}{EE_f}\frac{k_f}{k}\frac{m_B^2}{m_A^2}\sum_{slj}\frac{S_F D_0^2}{2s+1}\sum_m\sum_{l_f}\left|\sum_{l_i} T^{if}(\psi)\right|^2$$

$$\sigma_{J_f^\pi}(E) = \frac{8\pi(2J_f+1)S_F}{(2J_A+1)(2J_a+1)}\sum_{I_f,J_i,l_i,I_i}\left\{\frac{2k_\gamma^3}{9}\left(\left|\mathrm{M}_{E1}(\psi)\right|^2+\left|\mathrm{M}_{M1}(\psi)\right|^2\right)+\frac{k_\gamma^5}{150}\left|\mathrm{M}_{E2}(\psi)\right|^2\right\}$$

$$\{\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}+\frac{2\mu}{\hbar^2}[E-V_c(r)-V_n(r)]\}\psi=0$$

$$V_n(r)=\frac{V_R}{1+\exp[(r-R_R)/a_R]}+i\frac{V_S\exp[(r-R_S)/a_S]}{1+\exp[(r-R_S)/a_S]}$$

$\psi=\phi_{nL}(r)$ for bound state.

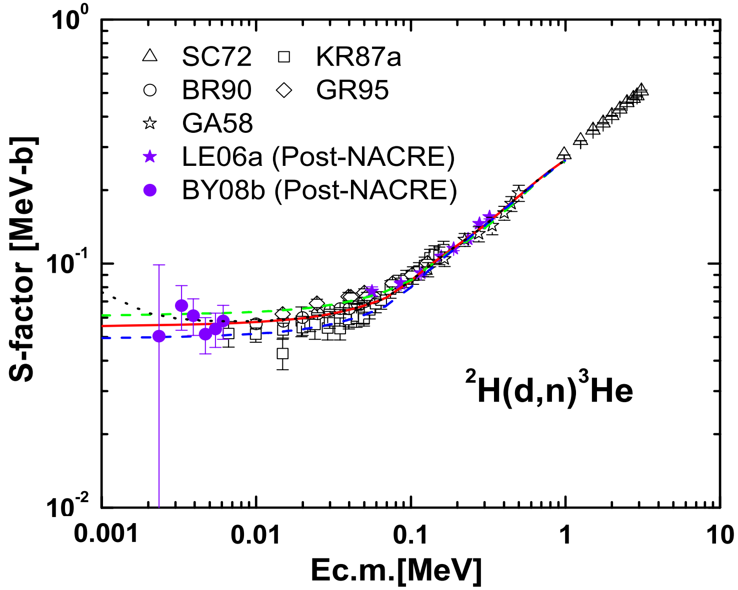
$\psi=\chi_L(k,r)$ for scattering state.

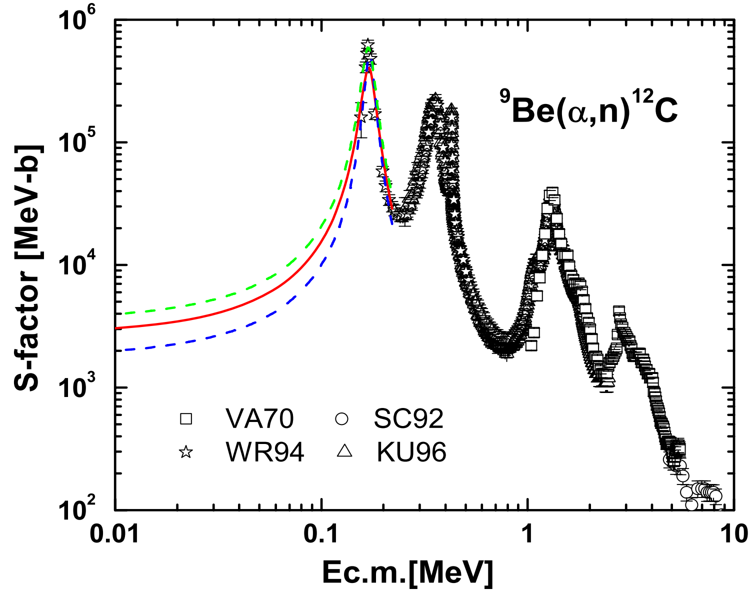
C. Fitting procedures: Transfer reaction

Determine the data range to be fitted

Case I: Data below $E_{c.m.}$ =1 MeV

Case II: Only the data for resonance at the lowest position of excitation energy





Fitting method and uncertainties estimation

Obtain several sets of results (parameters and extrapolations) by χ^2 fit to reproduce experimental data

$$\boxed{\{V, r, a\} \rightarrow V_n(r) \rightarrow \psi \rightarrow T^{if}, S_F D_0^2 \rightarrow \sigma_{trans}(E) \rightarrow \text{S-factors}(E)}$$

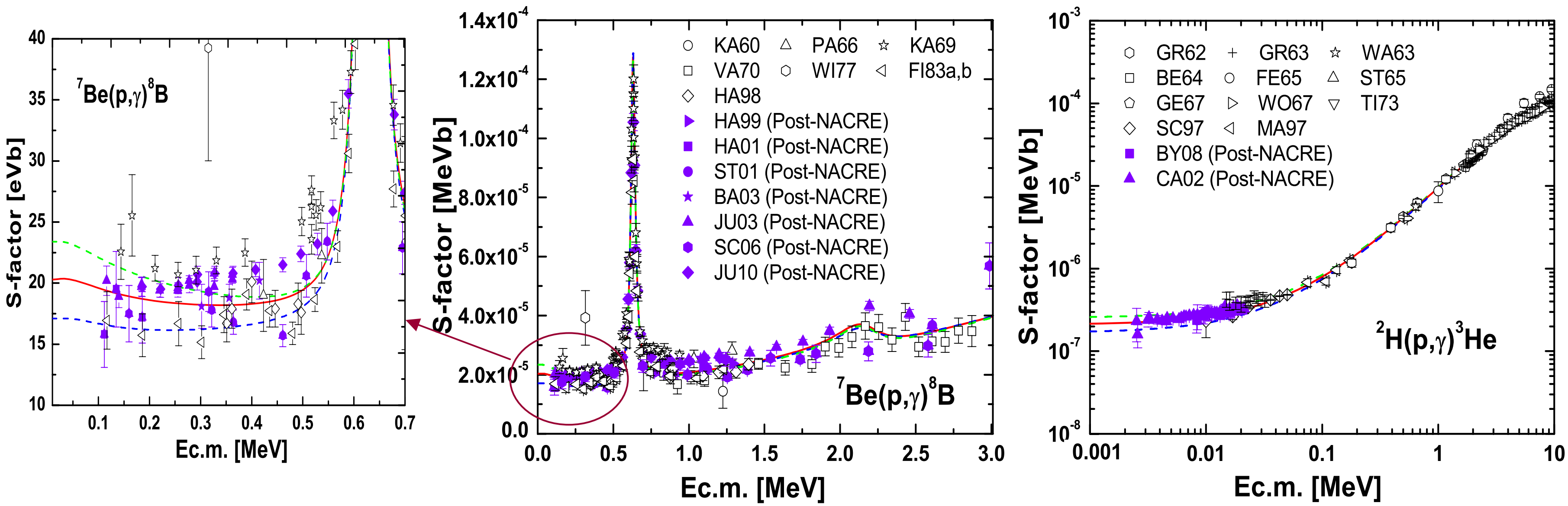
Choose the "adopted" "low" and "high" extrapolations as recommended results

Occasionally fit-by-eyes for some extrapolations

C. Fitting procedures: Radiative capture reaction

Determine the data range to be fitted

In general, we fit the resonances and non-resonant contributions simultaneously, and try to include all selected experimental data.



Fitting method and uncertainties estimation

Fit each resonance simultaneously when found experimentally

Add the non-resonant contributions to reproduce the total S-factors

Obtain several sets of results (parameters and extrapolations) by eyes-fit

$$\boxed{\{V, r, a\} \rightarrow V_n(r) \rightarrow \psi \rightarrow \mathrm{M}_{E1}, \mathrm{M}_{M1}, \mathrm{M}_{E2}, S_F \rightarrow \sigma_{cap}(E) \rightarrow \text{S-factors}(E)}$$

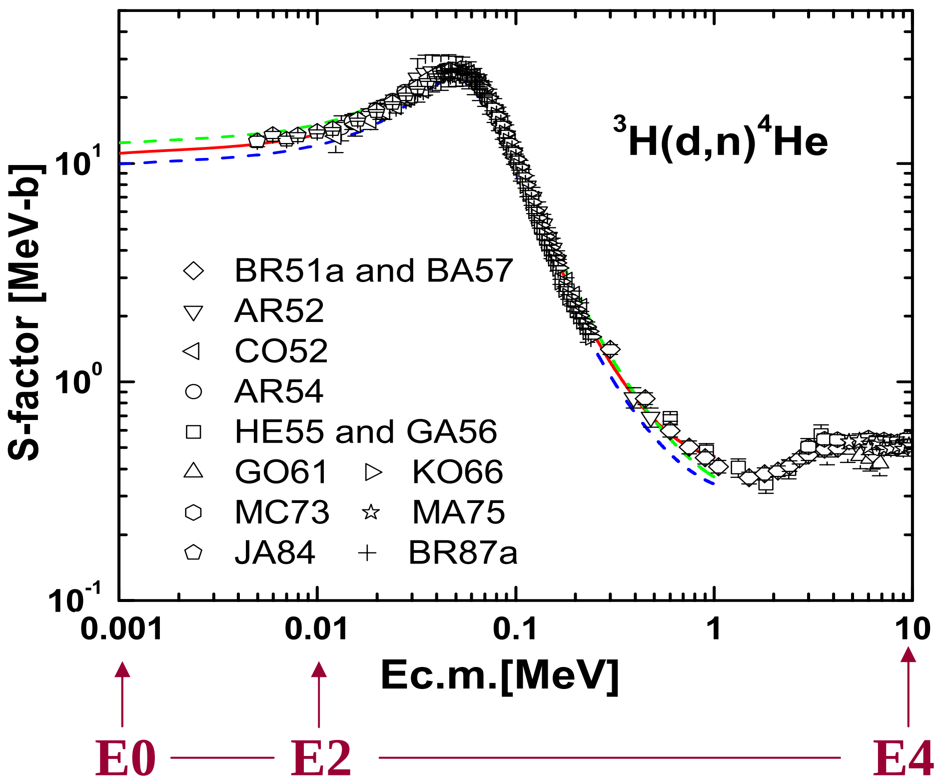
Choose the “adopted” “low” and “high” extrapolations as recommended results

D. Reaction rates evaluation

Thermonuclear reaction rate for A(a,b)B:

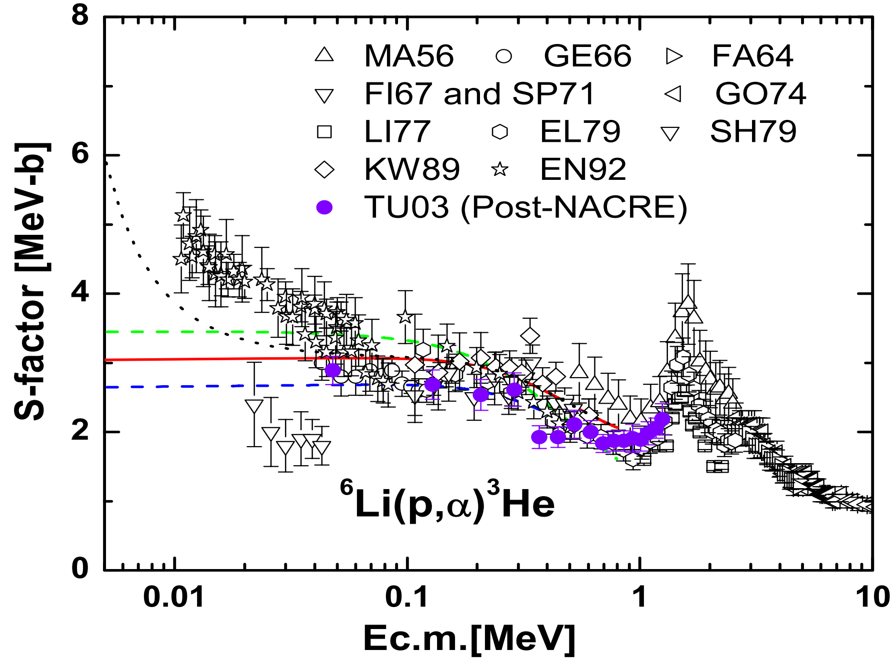
$$N_A < \sigma v >= N_A \frac{(8/\pi)^{1/2}}{\mu_\alpha^{1/2}(k_B T)^{3/2}} \int_0^\infty S(E) \exp[-2\pi\eta - E/(k_B T)] dE$$

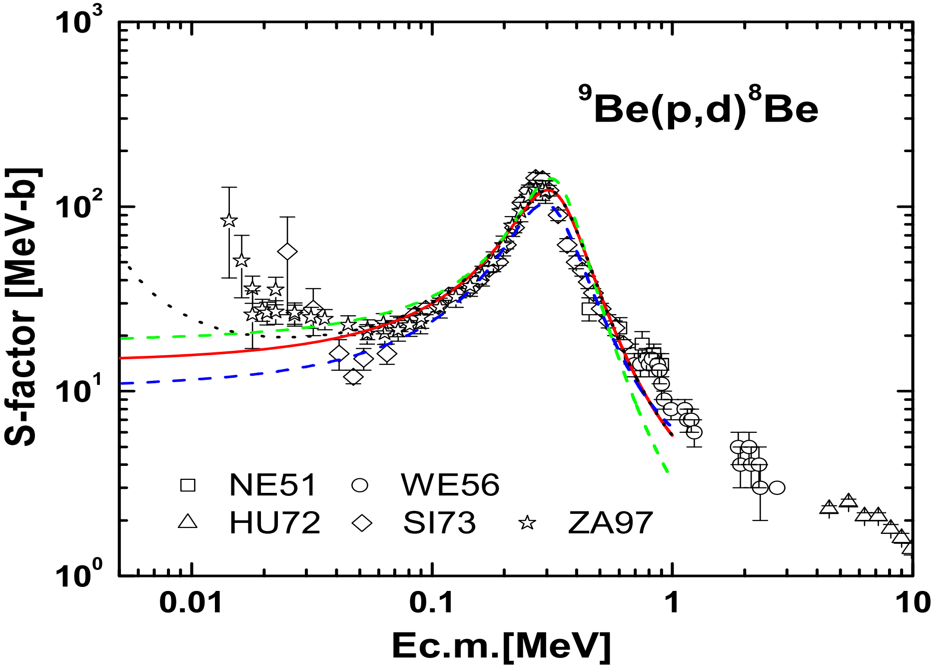
Reaction rates with uncertainties are evaluated from S-factors in two energy ranges:
(1) experimental data with error-bars when available in [E2,E4];
(2) "adopted" "low" and "high" extrapolations in [E0,E2].

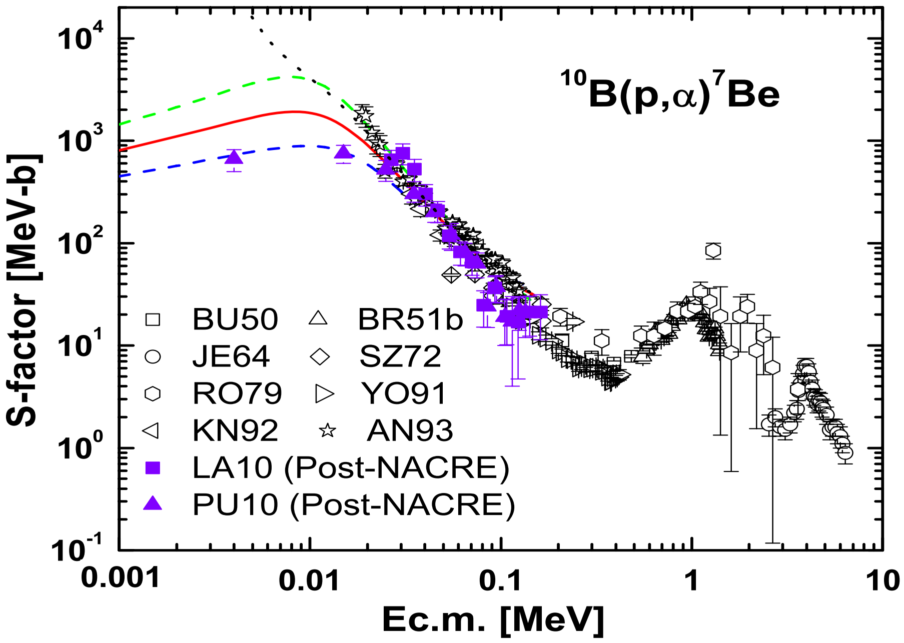


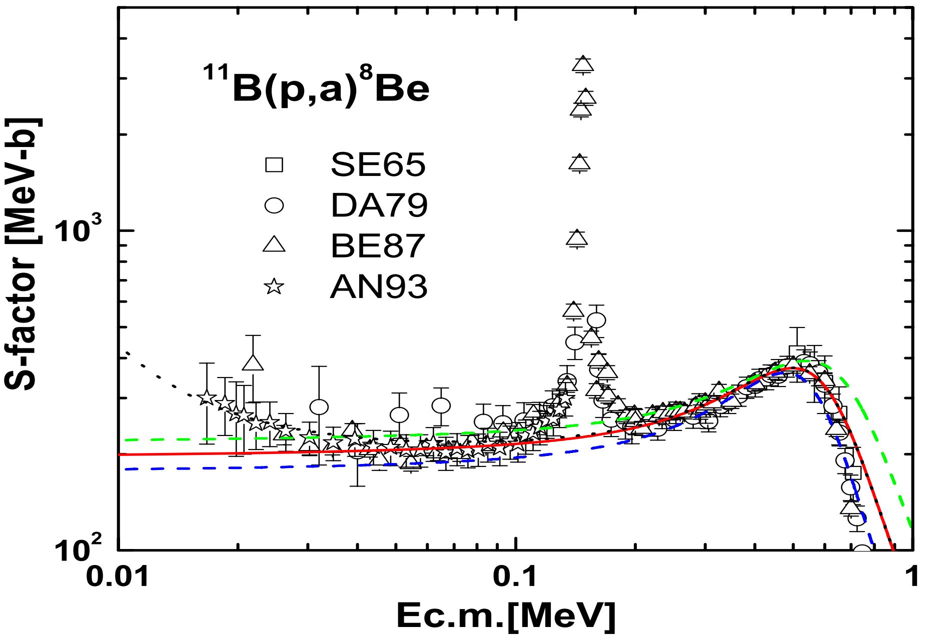
Thermonuclear reaction rates (adopted, low and high limits) are presented in the temperature range of [0.001, 10] GK in the tabular form.

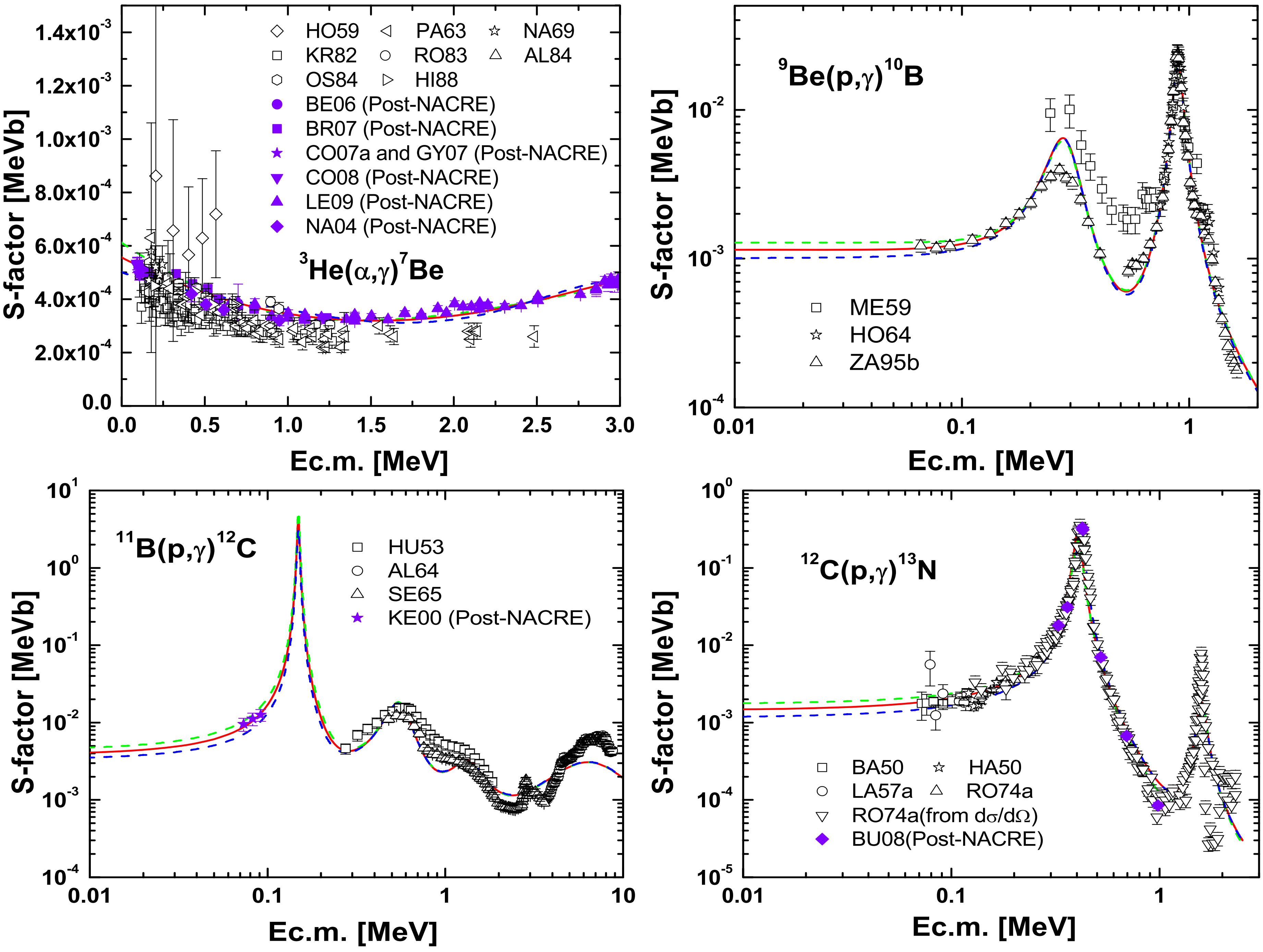
3. Results of Individual Reaction





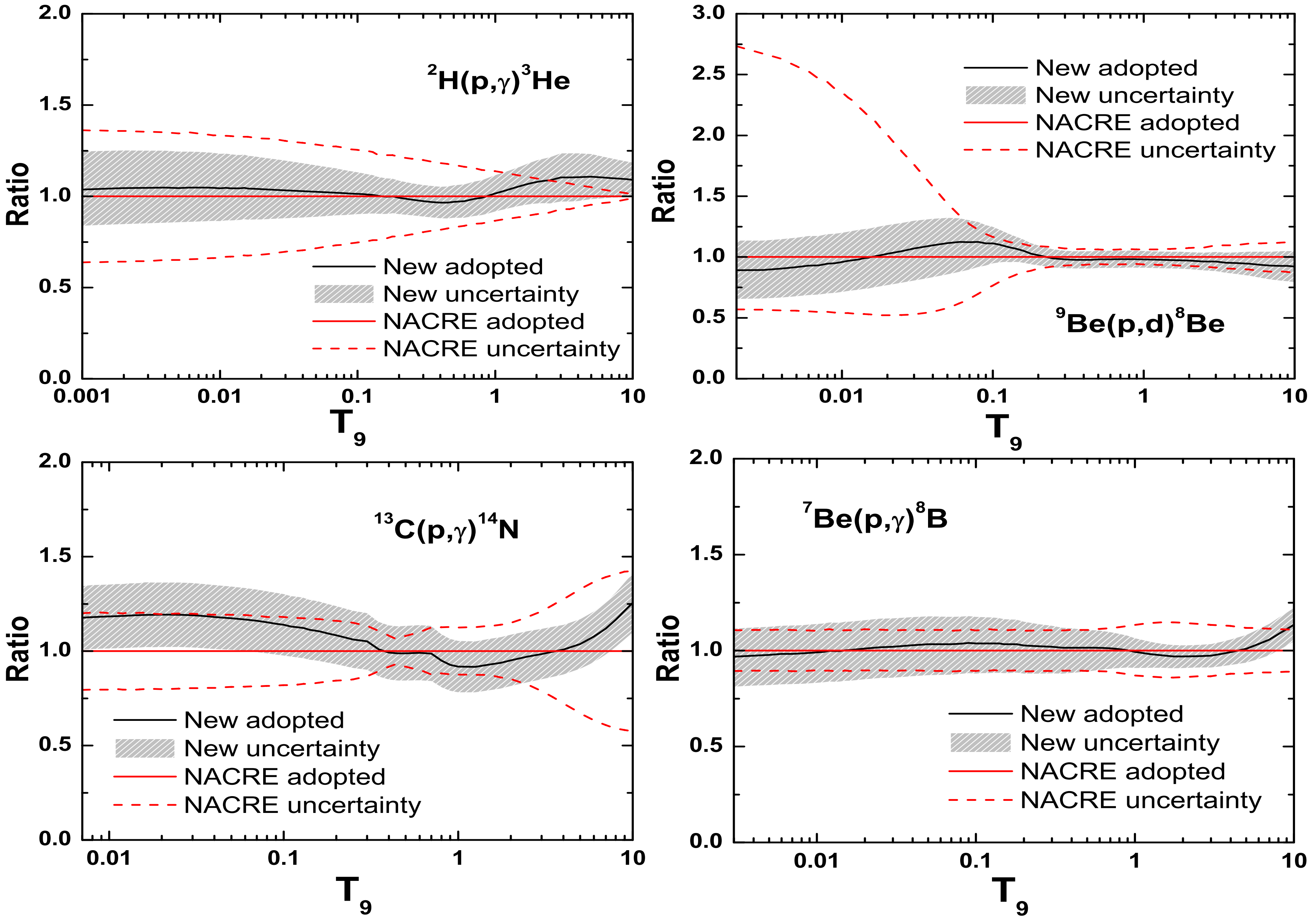






S-factor [MeVb]
Ec.m. [MeV]
^{3}He(α,γ)^{7}Be
HO59
PA63
NA69
KR82
RO83
AL84
OS84
HI88
BE06 (Post-NACRE)
BR07 (Post-NACRE)
CO07a and GY07 (Post-NACRE)
CO08 (Post-NACRE)
LE09 (Post-NACRE)
NA04 (Post-NACRE)
^{9}Be(p,γ)^{10}B
ME59
HO64
ZA95b
^{11}B(p,γ)^{12}C
HU53
AL64
SE65
KE00 (Post-NACRE)
^{12}C(p,γ)^{13}N
BA50
HA50
LA57a
RO74a
RO74a(from dσ/dΩ)
BU08(Post-NACRE)

Reaction rate comparison between NACRE II and NACRE



S(0) comparison between NACRE II and various compilations

Reaction	NACRE II	[1]	[2]	[3]	Reaction	NACRE II	[1]	[2]	[3]
$^{2}H(p,\gamma)^{3}He$	$0.214^{+0.04}_{-0.04}$	$0.223^{+0.01}_{-0.01}$	0.14	$0.214^{+0.017}_{-0.016}$	$^{2}H(d,n)^{3}He$	$55.53^{+5.90}_{-6.01}$	$52.4^{+3.50}_{-3.50}$	---	---
$^{2}H(d,p)^{3}H$	$56.20^{+4.88}_{-4.69}$	$57.1^{+0.80}_{-0.80}$	---	---	$^{3}H(d,n)^{4}He$	$11.13^{+1.27}_{-1.20}$	$11.7^{+0.20}_{-0.20}$	---	---
$^{3}H(\alpha,\gamma)^{7}Li$	$93.2^{+13.8}_{-14.4}$	$95.0^{+5.0}_{-5.0}$	---	---	$^{3}He(d,p)^{4}He$	$5.86^{+0.56}_{-0.53}$	$5.90^{+0.30}_{-0.30}$	---	---
$^{3}He(\alpha,\gamma)^{7}Be$	$0.557^{+0.055}_{-0.055}$	$0.51^{+0.04}_{-0.04}$	---	$0.56^{+0.03}_{-0.03}$	$^{6}Li(p,\gamma)^{7}Be$	$93.8^{+20.2}_{-18.6}$	---	99.5	---
$^{7}Li(p,\gamma)^{8}Be$	$1.27^{+0.91}_{-0.22}$	---	0.238	---	$^{7}Li(p,\alpha)^{4}He$	$57.69^{+8.81}_{-8.74}$	67^{+4}_{-4}	---	---
$^{7}Be(p,\gamma)^{8}B$	$19.9^{+3.2}_{-3.2}$	---	19.4	$20.8^{+1.6}_{-1.6}$	$^{9}Be(p,\gamma)^{10}B$	$1.15^{+0.13}_{-0.14}$	---	1.05	---
$^{12}C(p,\gamma)^{13}N$	$1.44^{+0.29}_{-0.28}$	---	2.346	$1.34^{+0.21}_{-0.21}$	$^{13}C(p,\gamma)^{14}N$	$8.13^{+1.19}_{-1.18}$	---	6.217	$7.6^{+1.0}_{-1.0}$
$^{13}N(p,\gamma)^{14}O$	$3.84^{+1.43}_{-1.07}$	---	5.771	---	$^{14}N(p,\gamma)^{15}O$	$1.80^{+0.46}_{-0.49}$	---	1.47	$1.66^{+0.12}_{-0.12}$
$^{15}N(p,\gamma)^{16}O$	$37.2^{+15.0}_{-10.2}$	---	22.1	36^{+6}_{-6}	$^{15}N(p,\alpha)^{12}C$	$66.6^{+13.2}_{-13.6}$	---	---	73^{+5}_{-5}

[1] P. Descouvemont, A. Adahchour, C. Angulo, A. Coc, E. Vangioni-Flam, At. Data Nucl. Data Tables 88 (2004) 203-236.

[2] J. T. Huang, C. A. Bertulani, V. Guimaraes, At. Data Nucl. Data Tables 96 (2010) 828-847.

[3] E. G. Adelberger, et. al., Rev. Mod. Phys. 83 (2011), 195-245.

4. Summary

The NACRE II compilation includes:

(1) the collection of experimental data published in refereed journals till 2010, before and after NACRE.

(2) the theoretical adopted, low, and high cross sections (S-factors) extrapolated to very low energies based on ZRDWBA or PM.

(3) the evaluated reaction rates along with their uncertainties in the temperatures range [10^6, 10^{10}] K.

The improved reaction models used in NACRE II with respect to NACRE make the evaluated cross sections and reaction rates more reliable, especially at low energies and temperatures.

Paper of NACRE II coming soon…

Thank you for your attention

B. Nuclear reaction model: Transfer reaction

Transfer reaction A(a,b)B: Stripping: a=x+b, A+x=B. Pickup: a+x=b, A=B+x.

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{1}{4\pi} C_{\alpha,\beta} \frac{1}{EE_f} \frac{k_f}{k} \frac{m_B^2}{m_A^2} \sum_{slj} \frac{S_F D_0^2}{(2s+1)} \sum_m \left| \sum_{l_i,l_f} \sqrt{\frac{(2l_f+1)(l_f-|m|)!}{(l_f+|m|)!}} P_{l_f}^{|m|}(\cos\theta) T^{if} \right|^2$$

$$\sigma_{J_B^\pi}(E) = C_{\alpha,\beta} \frac{1}{EE_f} \frac{k_f}{k} \frac{m_B^2}{m_A^2} \sum_{slj} \frac{S_F D_0^2}{2s+1} \sum_m \sum_{l_f} \left| \sum_{l_i} T^{if} \right|^2$$

$$T^{if} = i^{m+|m|+l_i-l_f-l} < l_i l_f l \mid m0m >< l_i l_f l \mid 000 > \frac{\left((2l_i+1)(2l_f+1)\right)^{1/2}}{(2l+1)} \int \chi_{l_f}\left(k_f, \frac{m_A}{m_B} r\right) \frac{\phi_{nl}^{js}(r)}{r} \chi_{l_i}(k,r)\, dr$$

$C_{\alpha,\beta}$ $\begin{cases} \text{Stripping: } (2J_b+1)/(2J_a+1) \\ \text{Pickup: } (2J_B+1)/(2J_A+1) \end{cases}$

Triangular relations hold: ***l=j-s, j=J_B-J_A, s=J_a-J_b***

E, k, l_i, χ_{li}, E_f, k_f, l_f, χ_{lf}: C. M. energies, wave numbers, orbital angular momentum, distorted scattering sates for entrance and exit channels.

J_A, J_a, J_b, J_B: spins of the four participating nuclei A, a, b and B.

s, l, j: spin, orbital and total angular momentum for transfer form factor.

m_A, m_B: masses of nucleus A and B.

S_F: spectroscopic factor. D_0: zero-range interaction strength.

P_L^M: associated Legendre polynomial.

$\phi(r)/r$: radial form factor bound in B or A.

B. Nuclear reaction model: Radiative capture reaction

Radiative capture reaction A(a, γ)B: Transition from the initial scattering state A+a forms the nucleus B with accompanying γ -ray emission. The E1, M1, and E2 transitions are considered.

$$\sigma_{J_f^\pi}(E)=\frac{8\pi(2J_f+1)C^2S_F}{(2J_A+1)(2J_a+1)}\sum_{I_f,J_i,l_i,I_i}\left\{\frac{2}{9}k_\gamma^3\left(\left|\mathrm{M}_{E1}\right|^2+\left|\mathrm{M}_{M1}+\mathrm{M}_{M1}^{\mathrm{int}}(A)+\mathrm{M}_{M1}^{\mathrm{int}}(a)\right|^2\right)+\frac{1}{150}k_\gamma^5\left|\mathrm{M}_{E2}+\mathrm{M}_{E2}^{\mathrm{int}}(A)+\mathrm{M}_{E2}^{\mathrm{int}}(a)\right|^2\right\}$$

$$\mathrm{M}_{E\lambda}=e\left[Z_A\left(\frac{m_a}{m_A+m_a}\right)^\lambda+Z_a\left(\frac{-m_A}{m_A+m_a}\right)^\lambda\right]\delta I_i I_f C_\lambda^{if}<l_i\lambda l_f\,|\,000>\mathrm{I}_\lambda^{if}$$

$$\mathrm{M}_{E\lambda}^{\mathrm{int}}(A)=\sqrt{5/4}\,Q_{2,A}\delta_{l_il_f}D_\lambda^{if}\mathrm{I}_0^{if}$$

$$\mathrm{M}_{M1}^{\mathrm{int}}(A)=\sqrt{3}\,\mu_{1,A}\delta_{l_il_f}D_1^{if}\mathrm{I}_0^{if}$$

$$\mathrm{M}_{M1}=\mu_N\frac{Z_Am_a^2+Z_am_A^2}{m_Am_a(m_A+m_a)}\delta_{I_iI_f}\delta_{l_il_f}\left(-C_1^{if}\right)\left[l_i(l_i+1)\right]^{1/2}\mathrm{I}_0^{if}$$

$$\mathrm{I}_\nu^{if}=\int\phi_{nl_f}(r)\,r^\nu\chi_{l_i}(E,r)\,dr$$

$$D_\lambda^{if}=(-)^{J_A+J_a-I_f-l_f}\sqrt{\frac{(2J_i+1)(2J_A+1)(2I_i+1)(2I_f+1)}{4\pi}}\begin{Bmatrix}J_i & J_f & \lambda\\ I_f & I_i & l_f\end{Bmatrix}\begin{Bmatrix}J_A & J_a & I_f\\ I_i & \lambda & J_A\end{Bmatrix}/<J_A\lambda J_A\,|\,J_A0J_A>$$

$$C_\lambda^{if}=(-)^{l_i+J_i+I_i}\sqrt{\frac{(2\lambda+1)(2J_i+1)(2l_i+1)}{4\pi}}\begin{Bmatrix}J_i & J_f & \lambda\\ l_f & l_i & I_i\end{Bmatrix}$$

Triangular relations hold: $\mathbf{J_i}$=$\mathbf{J_f}$- λ , $\mathbf{l_i}$=$\mathbf{l_f}$- λ ($\mathrm{M}_{E\lambda}$, M_{M1}), **Ii=If**- λ ($\mathrm{M}^{\mathrm{int}}_{E\lambda}$, $\mathrm{M}^{\mathrm{int}}_{M1}$)

I, l, J, π : channel spin, orbital angular momentum, total angular momentum, parity.

Indexes i, f, a, A, B: index of initial state, final state, nucleus a, A, B.

λ : multi-polarity. k γ : photon wave number.

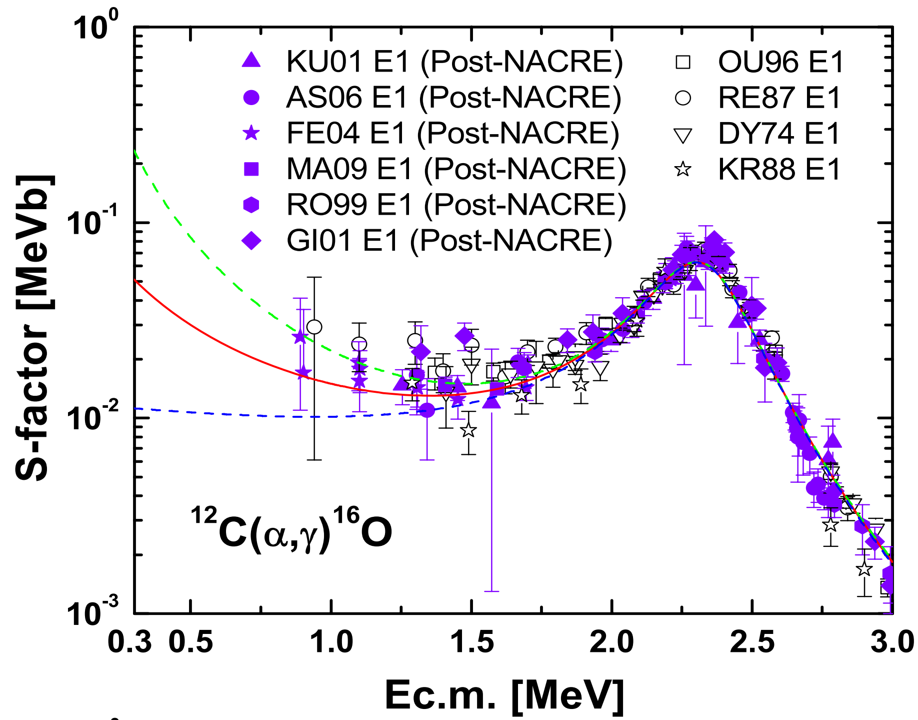
C^2S_F: the spectroscopic factor including the isospin CG coefficient C.

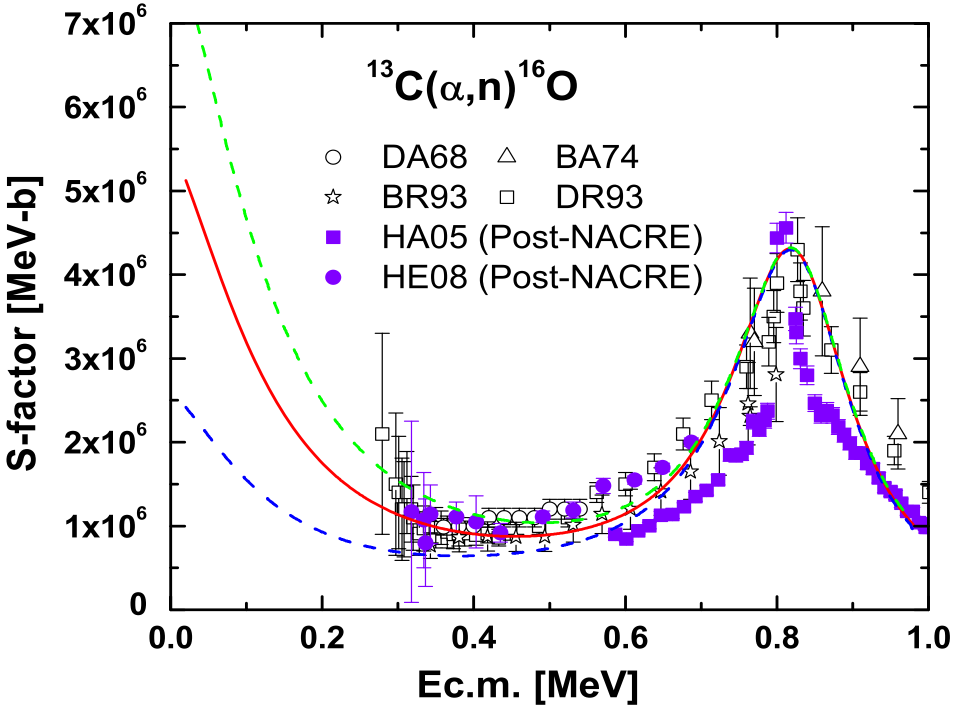
Q_2, μ_1: electric quadrupole moment, magnetic dipole moment.

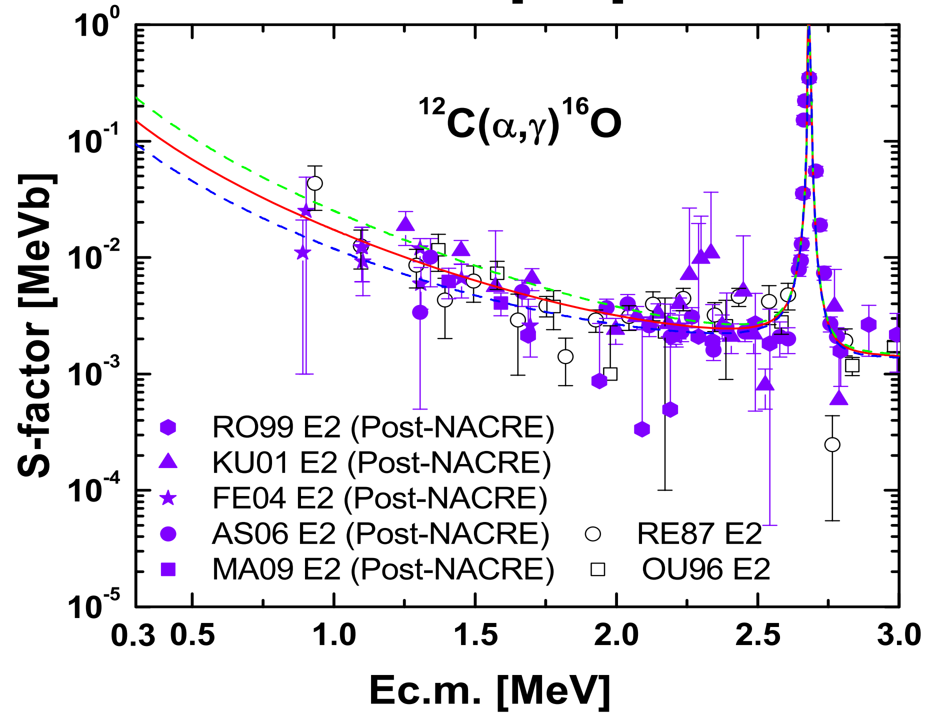
Possible contributions from sub-threshold resonances

$^{12}C(\alpha,\gamma)^{16}O$ and $^{13}C(\alpha,n)^{16}O$

To scrutinize the sub-threshold feature by nuclear structure information is beyond the scope and capability of this compilation using the simple potential models. We merely empirically seek for S-factor enhancements that may be permissible on top of the available data with an allowance for a contribution additional to those derived by our standard fit procedure.







Transition to exited final nuclear states

$^{13}C(p,\gamma)^{14}N$ and $^{14}N(p,\gamma)^{15}O$

Experimentally, the cross sections for a few individual states fed by primary gamma-rays have been measured. Therefore, we analyze the partial cross sections corresponding to transition to individual final exited states, separately. The total cross sections are obtained by summing up all the partial cross sections.