Workshop on
Thermonuclear Reaction Rates for Astrophysics Applications
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Theoretical predictions of reaction rates for light systems

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   - Microscopic cluster models
3. **Applications**
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   - The $^{2}H(d,p)^{3}H$, $^{2}H(d,n)^{3}He$, and $^{2}H(d,\gamma)^{4}He$ reactions
4. **Conclusions**

1. INTRODUCTION

General problems in nuclear astrophysics

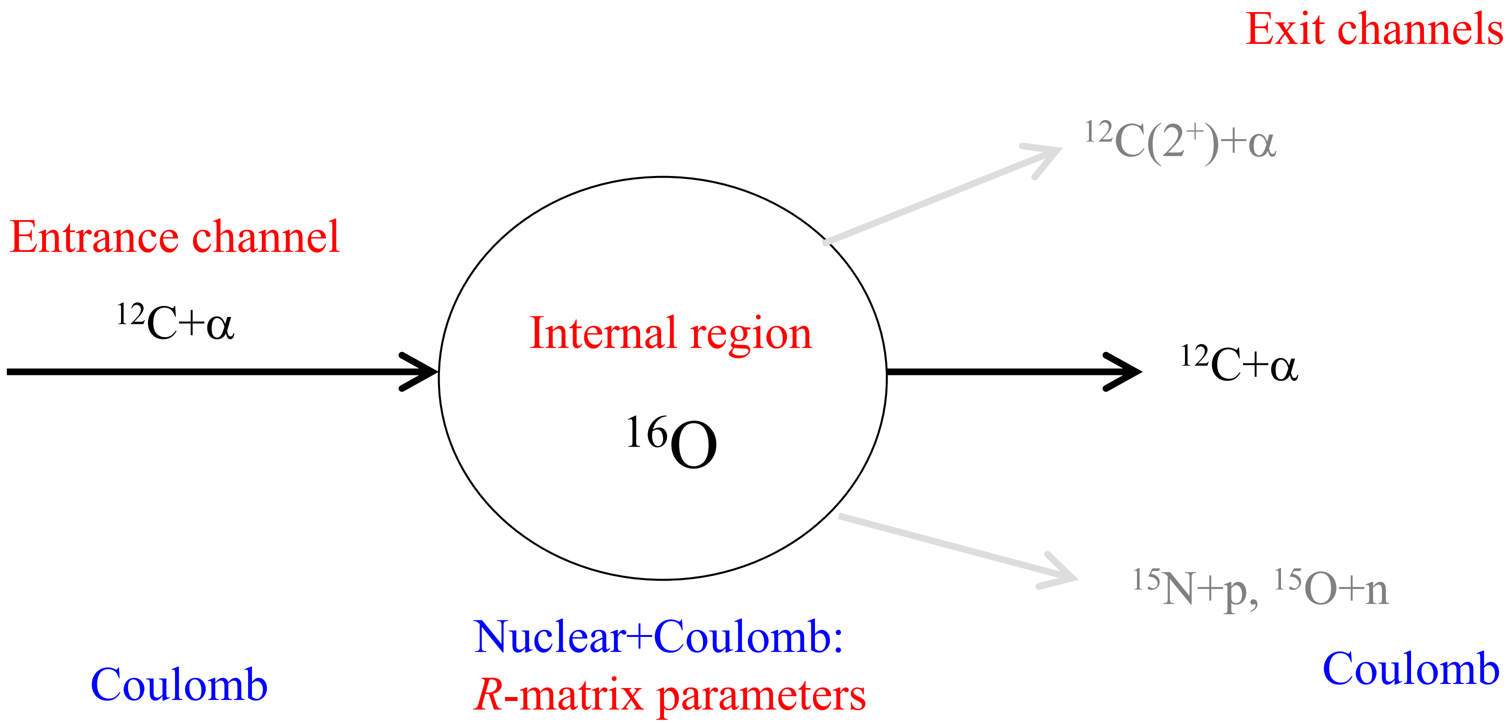
- Low energies → very low cross sections (Coulomb barrier)
- For heavy nuclei: high level densities → many resonances must be known
- Need for radioactive beams
- No systematics (many different types of reactions)
- transfer, capture
- resonant, non-resonant
- low or high level densities

➔ in most cases a theoretical support is necessary

- **data extrapolation** (example: **R-matrix method**)
  Available cross sections are parametrized, and extrapolated down to stellar energies
- **determination of cross sections**
  The cross sections are determined from the wave functions of the system
  No need for experimental data (in principle!)
  Examples: potential model, **microscopic models** (low level densities)
  shell model (resonance properties in for high level densities)

The R-matrix method

- Introduced by Wigner (1937) to parametrize resonances (nuclear physics)
- Provides scattering properties at all energies (not only at resonances)
- Based on the existence of 2 regions (radius a):
  - Internal: coulomb+nuclear
  - external: coulomb

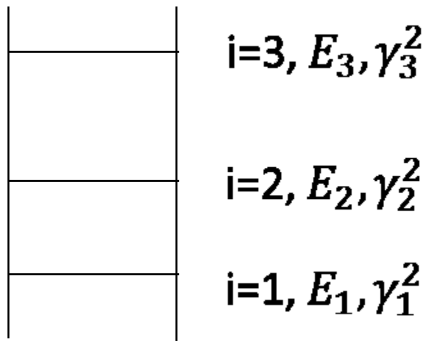


2. MODELS: the R-matrix method

- **Internal region: The R matrix is given by a set of resonance parameters E_i, γ_i^2**

$$R(E) = \sum_i \frac{\gamma_i^2}{E_i - E}$$

$$= a\frac{\Psi'(a)}{\Psi(a)}$$



- **External region: Coulomb behaviour of the wave function**

$$\Psi(r) = C\big(I(r) - UO(r)\big)$$

→the collision matrix U is deduced from the R-matrix (repeated for each spin/parity $J\pi$)

- **Two types of applications:**
  - **phenomenological R matrix: γ_i^2 and E_i are fitted to the data (astrophysics)**
  - **calculable R matrix: γ_i^2 and E_i are computed from basis functions (scattering theory)**

- **Can be extended to multichannel calculations (transfer), capture, etc.**

- **Well adapted to nuclear astrophysics: low energies, low level densities**

Microscopic cluster models

- **Goal: solution of the Schrödinger equation $H\Psi = E\Psi$**

with Hamiltonian: $H = \sum_i T_i + \sum_{j>i} V_{ij}$

T_i = kinetic energy of nucleon *i*

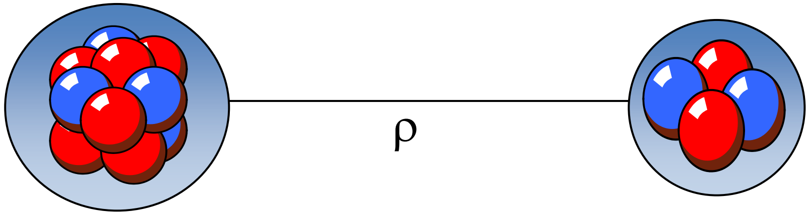
V_{ij} = nucleon-nucleon interaction (nuclear + coulomb)

- **Cluster approximation $\Psi = \mathcal{A}\phi_1\phi_2 g(\rho)$**

with ϕ_1, ϕ_2 = internal wave functions (input, shell-model)

$g(\rho)$ =relative wave function (output)

$\mathcal{A}$ = antisymmetrization operator

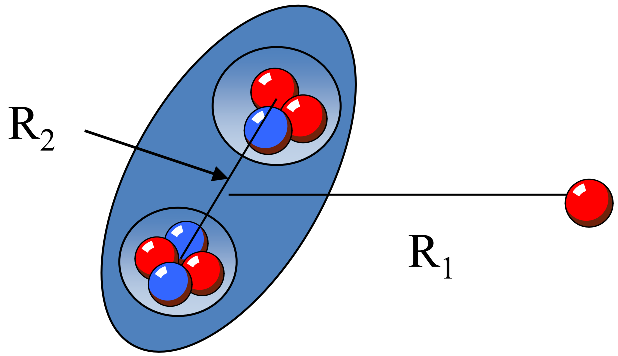


- **Can be applied to reactions *(E>0)* AND spectrocopy *(E<0)***
- **Cluster calculations: effective NN interactions**
- **Ab initio calculations: realistic NN interactions (limited to small nucleon numbers)**

Many applications: not only nuclear astrophysics
spectroscopy, exotic nuclei, elastic and inelastic scattering, etc.

Extensions:

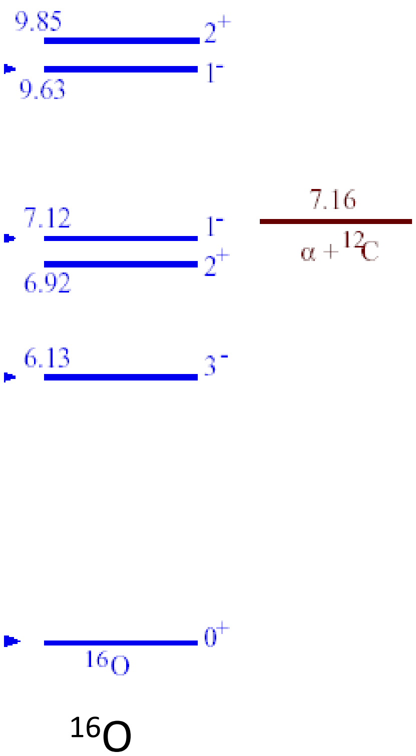
- **Multicluster calculations:** → deformed nuclei (example: ^{7}Be+p)



- **Multichannel calculations:** $\Psi = \mathcal{A}\phi_1\phi_2 g(\rho) + \mathcal{A}\phi_1^*\phi_2^* g^*(\rho) + \cdots$
  → better wave functions
  → inelastic scattering, transfer

- **Ab initio calculations:** no cluster approximation
  → very large computer times
  → limited to light nuclei
  → difficult for scattering (essentially limited to nucleon-nucleus)

Application 1: The $^{12}C(\alpha,\gamma)^{16}O$ reaction (E2 component) with the R-matrix



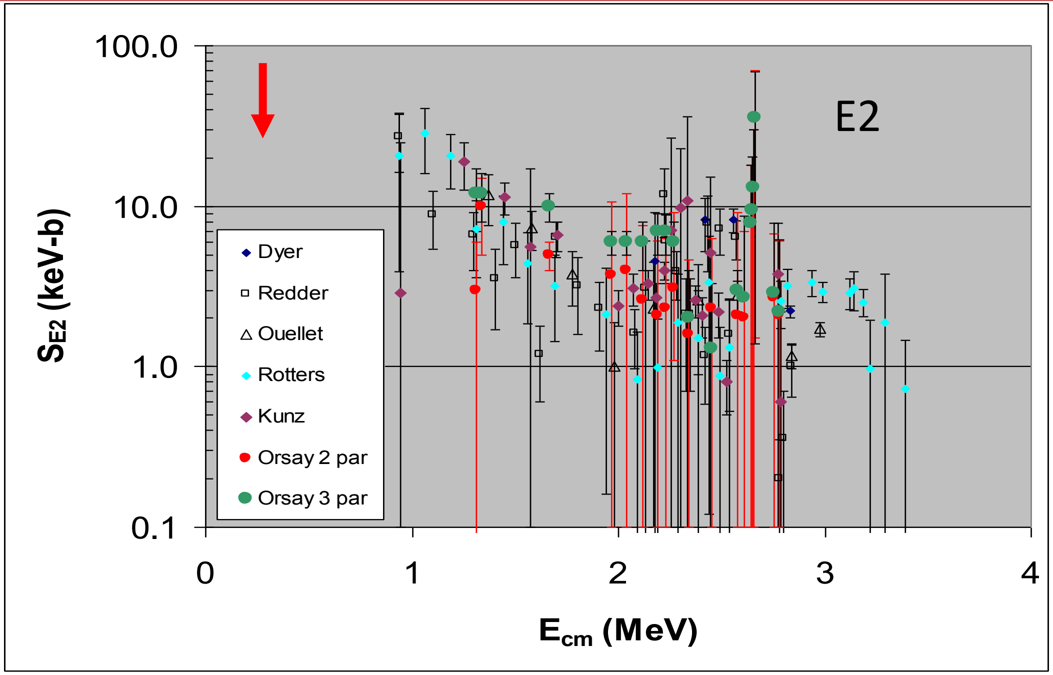
Important for the $^{12}C/^{16}O$ ratio

Many difficulties:

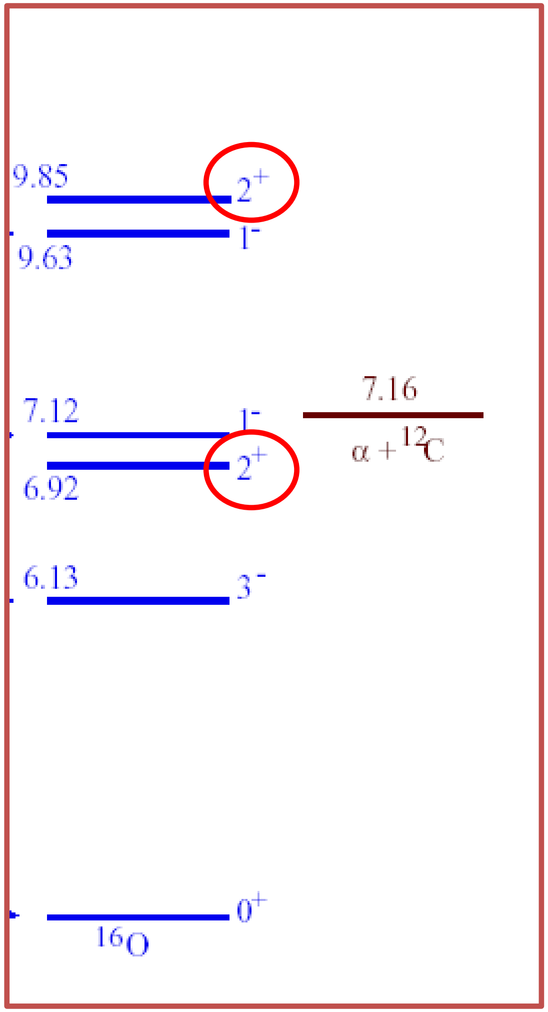
- E1 and E2 are important (E1 better known than E2)
- Interference effets
- Subthreshold states

→ many experiments, many calculations

→ extrapolation (0.3 MeV) very uncertain



3. Applications: the $^{12}C(\alpha,\gamma)^{16}O$ reaction



E2 → 2^+ states (=**poles**) are involved

- 2^+_1 subthreshold state
- 2^+_2, 2^+_3 narrow resonances at 2.68 and 4.36 MeV
- background (high energy): other resonances

« Observed » (experimental) data:

E_{ri}: energy of pole i

$\Gamma_{\alpha i}$: α width (reduced γ^2)

$\Gamma_{\gamma i}$: γ width

link

« R-matrix » data: $E_i, \gamma_i^2, \Gamma_{\gamma i}$

$$R(E) = \sum_{i=1}^{4} \frac{\gamma_i^2}{E_i - E} \rightarrow \text{phase shift}$$

$$\sigma(E) \sim \left| \sum_{i=1}^{4} \frac{\gamma_i \sqrt{\Gamma_{\gamma i}}}{E_i - E} \right|^2 \rightarrow \text{capture cross section}$$

3. Applications: the $^{12}C(\alpha,\gamma)^{16}O$ reaction

Ref: M. Dufour, P.D., Phys. Rev. C 78 (2008) 015808

4 poles, 3 parameters for each pole ➔ 12 parameters

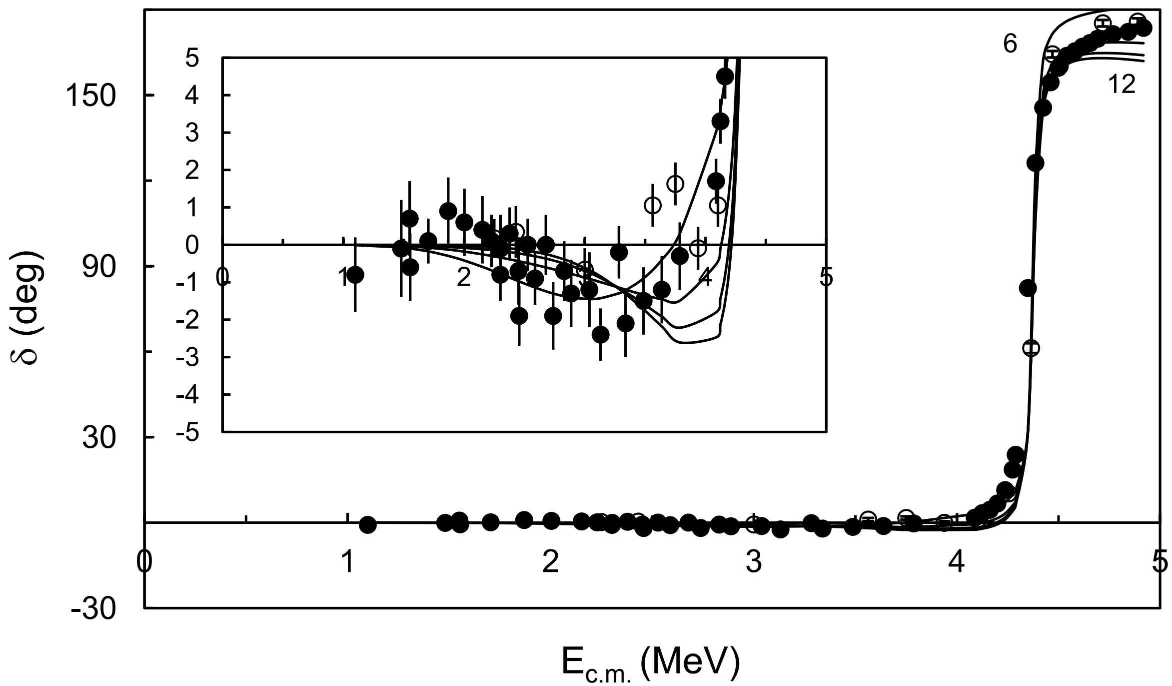
pole	E_i	γ^2_i	$\Gamma_{\gamma i}$
i=1	-0.24	fitted	exp.
i=2	2.68	exp.	exp.
i=3	4.36	exp.	exp.
i=4	variable	fitted	fitted

➔ 3 parameters (+background)

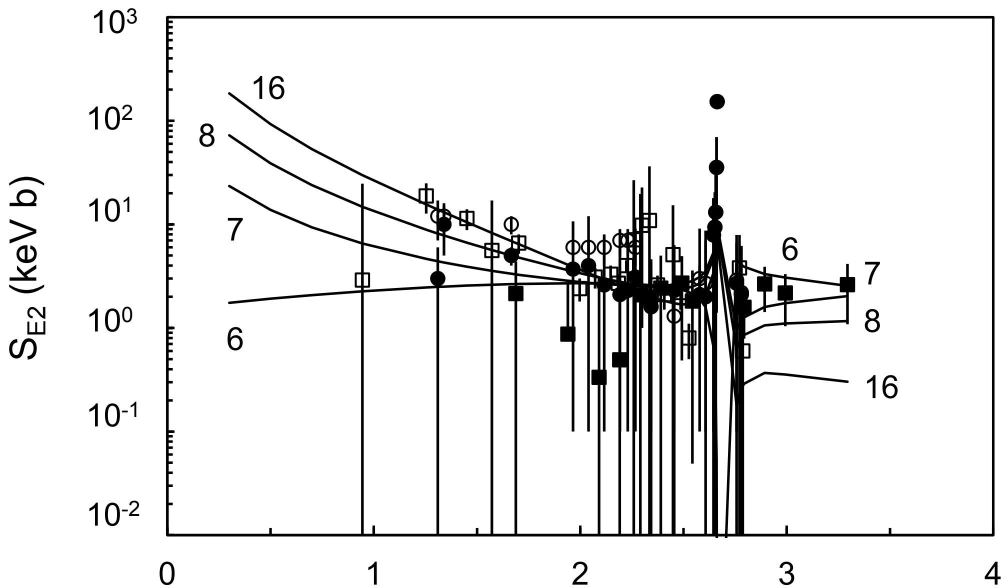
3 steps

1. Fix E_4 to different values
2. Fit of γ^2_1, γ^2_4 (on the 2^+ phase shift)
3. Fit of $\Gamma_{\gamma 4}$ (on the E2 S-factor)

Fit of the phase shifts → γ_1^2, γ_2^2
several values of the background energy E_4



Fit of the S factor



all χ^2 equivalent!

χ^2~3 ➔ no « good » fit

➔ only an upper limit can be deduced: S(300 keV) < 190 keV.b

➔ can we constrain the R-matrix fit with microscopic results?
yes, with the ANC of the 2^+ subthreshold state

Constrain from ANC

- Stands for **A**symptotic **N**ormalization **C**onstant
- Defined from the asymptotic properties of the radial wave function

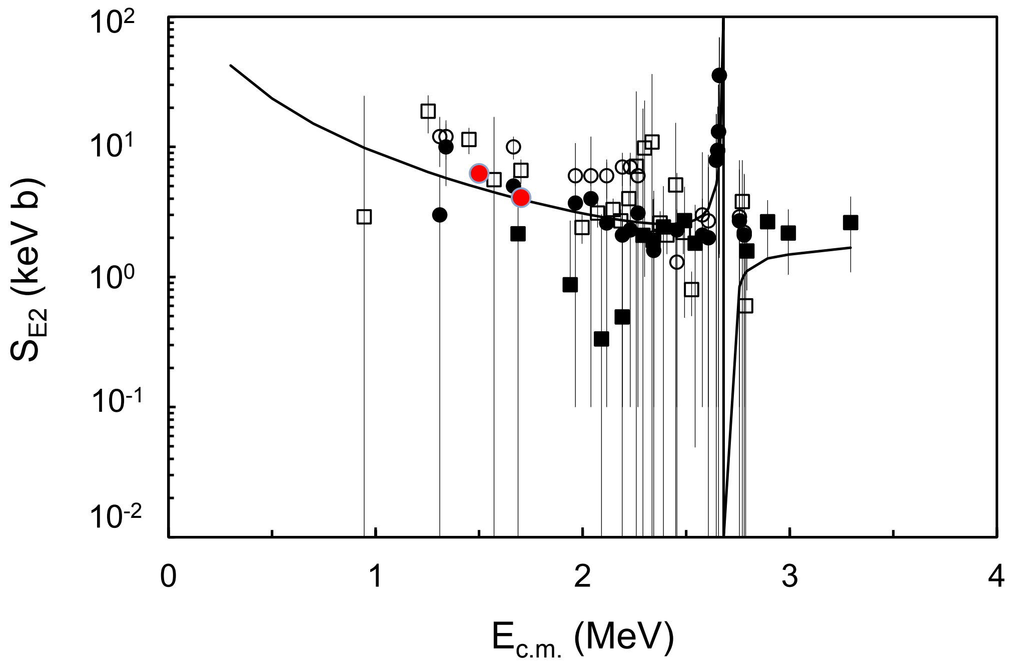
$$g_\ell(\rho) \longrightarrow C_\ell W_{-\eta_B,\ell+1/2}(2k_B\rho)$$ W(x)=Wittakher function

- ANC proportional to γ^2 ➔ for pole 1, γ^2 is taken from the theory
- Can be computed from microscopic models

pole	E_i	γ^2_i	$\Gamma_{\gamma i}$
i=1	-0.24	microscopic	exp.
i=2	2.68	exp.	exp.
i=3	4.36	exp.	exp.
i=4	variable	fitted	fitted

➔ only 2 parameters (+background energy)

3. APPLICATIONS: the $^{12}C(\alpha,\gamma)^{16}O$ reaction



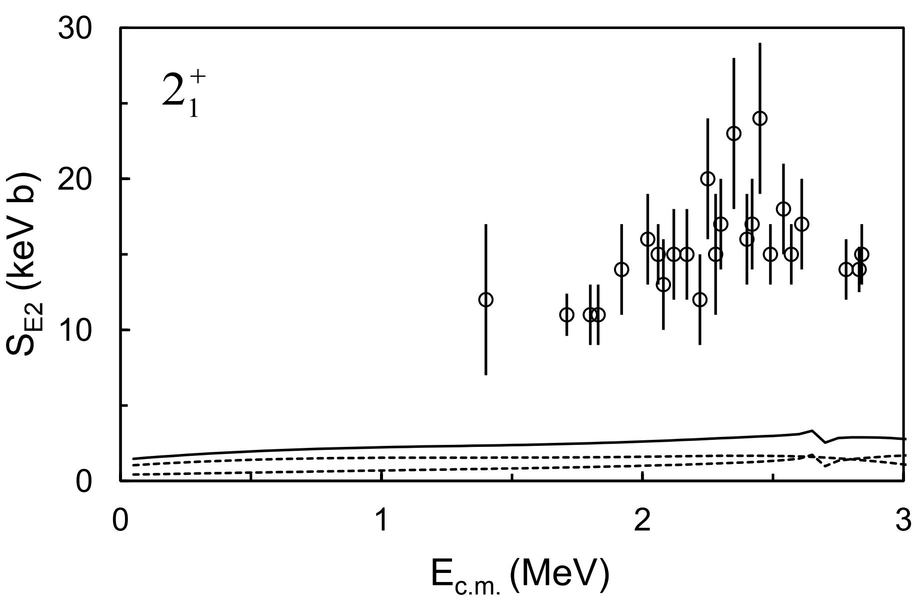
- Virtually independent of the background energy
- S_{E2}(300 keV)=42 ± 2 keV.b

→ ANC provides a strong constraint [cf C. Brune et al. PRL 83 (1999) 4025]

Discussion of the 2_1^+ ANC
Can be obtained from several techniques

- **indirect (transfer) measurements: C ~ 1.5×10^5 fm$^{-1/2}$**
  example $^{12}C(^6Li,d)^{16}O$
- **capture to 2^+ state (« cascade »): C ~ 3.2×10^5 fm$^{-1/2}$**
  2^+ weakly bound → external capture ($\sigma \sim |C|^2$)

- **present: C = 1.3×10^5 fm$^{-1/2}$**
  Microscopic cluster model

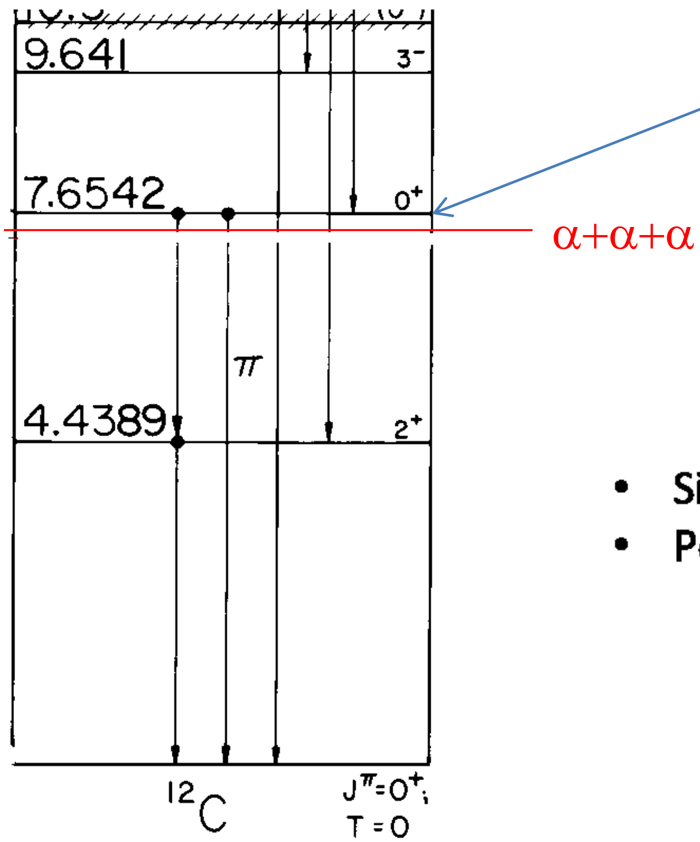


- capture to the 2_1^+ state (E=-0.24 MeV)
- external process ($\sigma \sim |C|^2$ at low E)
- 2.5 MeV resonance (E1) missing
- amplitude inconsistent with indirect measurements
- Data from Redder et al. NPA462 (1987) 385

→ new experiment??

Application 2: variation of the $^{12}C(0_2^+)$ state with the deuteron binding energy

- Goal: Effects of the variation of fundamental constants on Population III stellar evolution
- Ref: S. Ekström et al., A&A 514 (2010) A62



Hoyle state 0_2^+

Determines the triple α reaction rate

→ what happens if fundamental constants change?

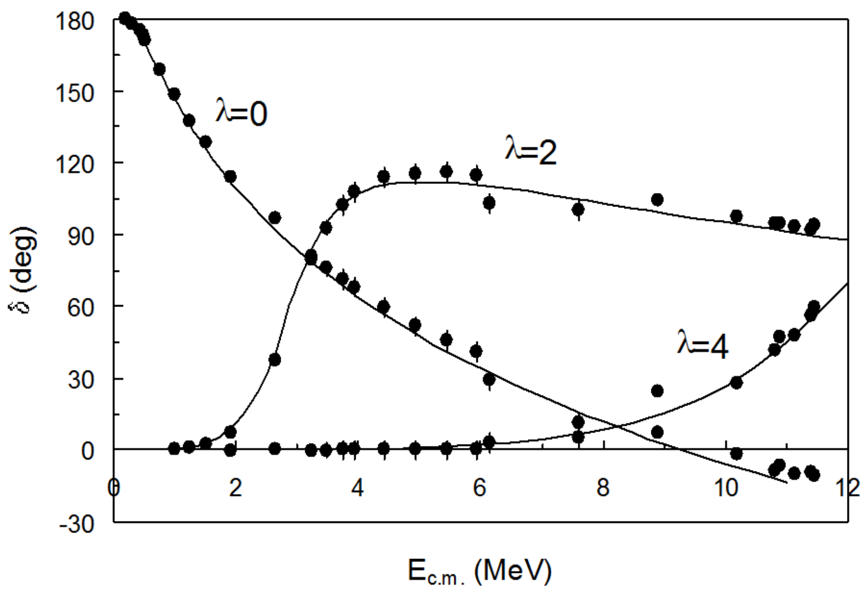
- Simultaneous description of d=p+n, $^8Be=\alpha+\alpha$, $^{12}C=\alpha+\alpha+\alpha$
- Possible in a microscopic model (NN interaction)

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j=1}^{A} V_{ij}$$

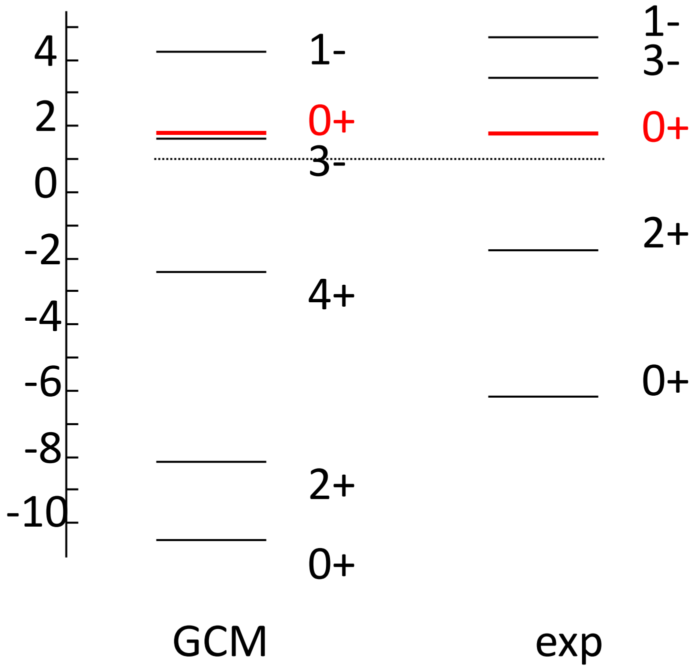
- **First calculation:** $V_{ij} = \frac{e^2}{|r_i - r_j|} + V_{ij}^N$

  V_{ij}^N =nucleon-nucleon interaction (Minnesota)
  contains one parameter (standard value u=1): adjusted on E(^{12}C)-E(^{8}Be)
  reproduces E(d)=-2.22 MeV (independent of u)

α–α phase shifts



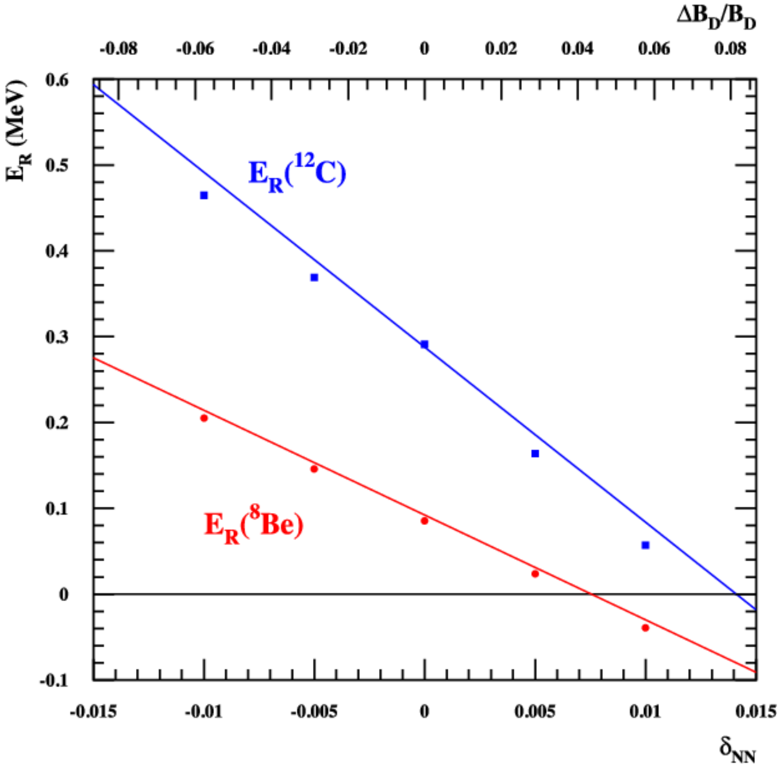
^{12}C



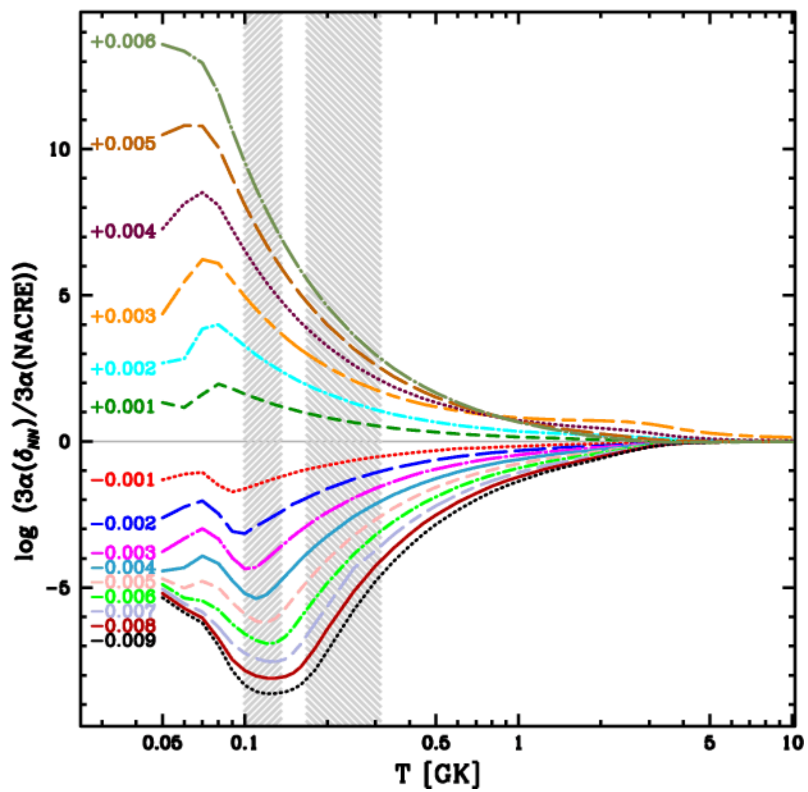
- **Second calculation:** $V_{ij} = \frac{e^2}{|r_i - r_j|} + (1 + \delta_{NN}) V_{ij}^N$

  Variation of δ_{NN} → d, ^{8}Be, ^{12}C change

  → variation of E(^{12}C)-E(^{8}Be) as a function of E(d)



Sensitivity of the reaction rate as a function of δ_{NN}



Application 3: The $^{2}H(d,p)^{3}H$, $^{2}H(d,n)^{3}He$, and $^{2}H(d,\gamma)^{4}He$ reactions

Differences between cluster and ab initio models

Cluster models

- In general a good approximation, but do not allow the use of realistic NN interactions
- Example: α particle described by four 0s orbitals

→ intrinsic spin =0

→ no spin-orbit, no tensor force, no 3-body force

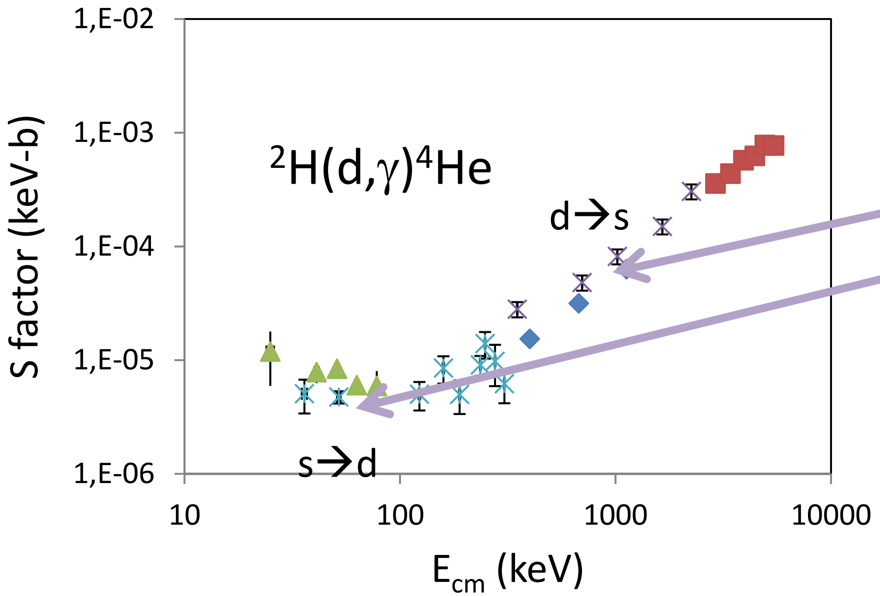
→ these terms are simulated by (central) NN interactions

Ab initio models

- No cluster approximation
- Use of realistic NN interactions (fitted on deuteron energy, Q, NN phase shifts, etc.)
- Application: d+d systems $^{2}H(d,\gamma)^{4}He$, $^{2}H(d,p)^{3}H$, $^{2}H(d,n)^{3}He$
- two physics issues
  - Analysis of the d+d S-factors (Big-Bang nucleosynthesis)
  - Role of the tensor force in $^{2}H(d,\gamma)^{4}He$

$^2H(d,\gamma)^4He$ S factor

- Ground state of ^4He=0+
- E1 forbidden→ main multipole is E2 → 2^+ to 0^+ transition→ d wave as initial state
- Experiment shows a plateau below 0.1 MeV: typical of an s wave
- Interpretation : the ^4He ground state contains an admixture of d wave
  final 0^+ state: $\Psi^{0+} = \Psi^{0+}(L=0,S=0) + \Psi^{0+}(L=2,S=2) = |0^+,0> + |0^+,2>$
  initial 2^+ state: $\Psi^{2+} = \Psi^{2+}(L=2,S=0) + \Psi^{2+}(L=0,S=2) = |2^+,0> + |2^+,2>$



E2 matrix element $< \Psi^{0+} | E2 | \Psi^{2+} >$

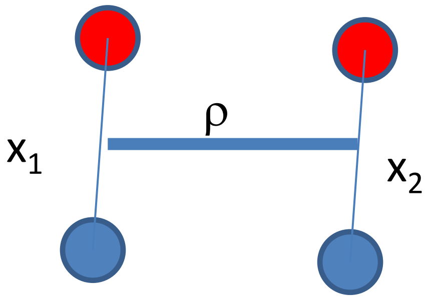
$\approx < 0^+,0 | E2 | 2^+,0 >$: d→s, dominant E>100 keV

$+ < 0^+,2 | E2 | 2^+,0 >$: s→d, tensor (E<100 keV)

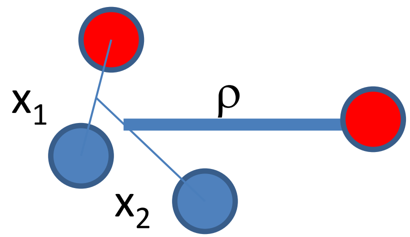
→ direct effect of the tensor force

Application: d+d systems

- Collaboration Niigata (K. Arai, S. Aoyama, Y. Suzuki)-Brussels (D. Baye, P.D.)
- Mixing of d+d, ^{3}H+p, ^{3}He+n configurations
- Ref.: K. Arai et al., Phys. Rev. Lett. 107 (2011) 132502



d+d

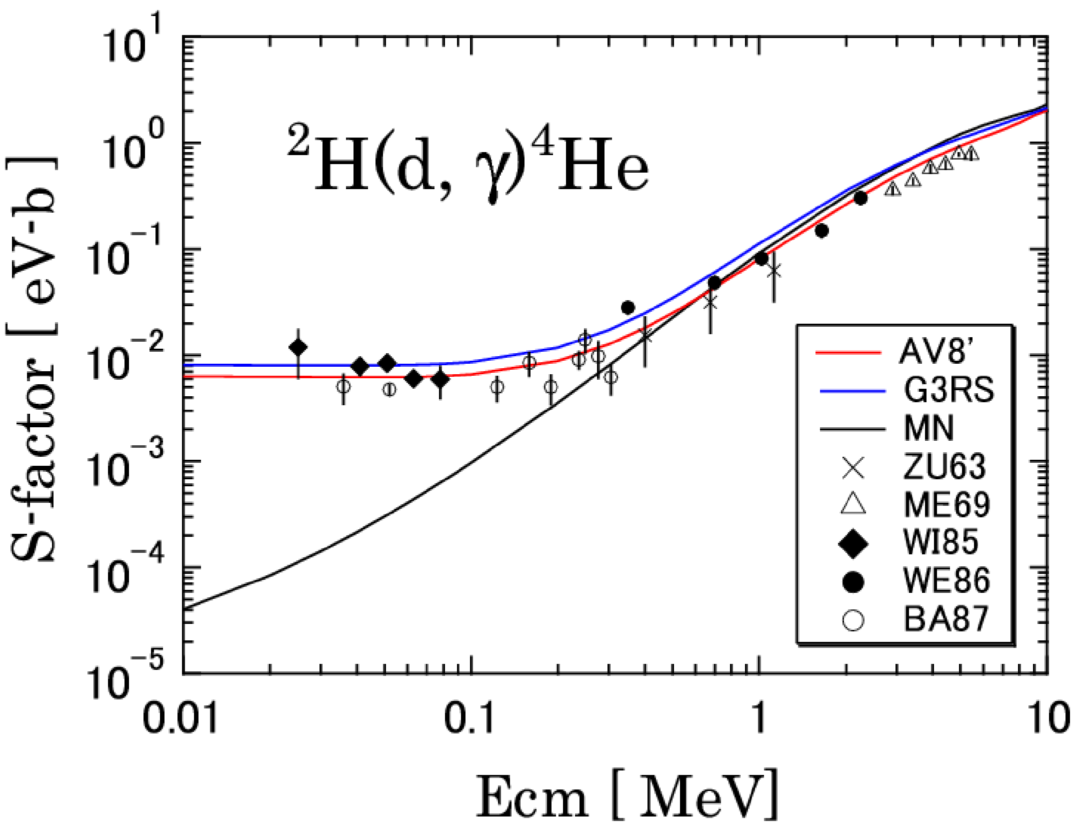


^{3}H+p, ^{3}He+n

- The total wave function is written as an expansion over a gaussian basis
- Superposition of several angular momenta
- 4-body problem (in the cluster approximation we would have: x_1=x_2=0)

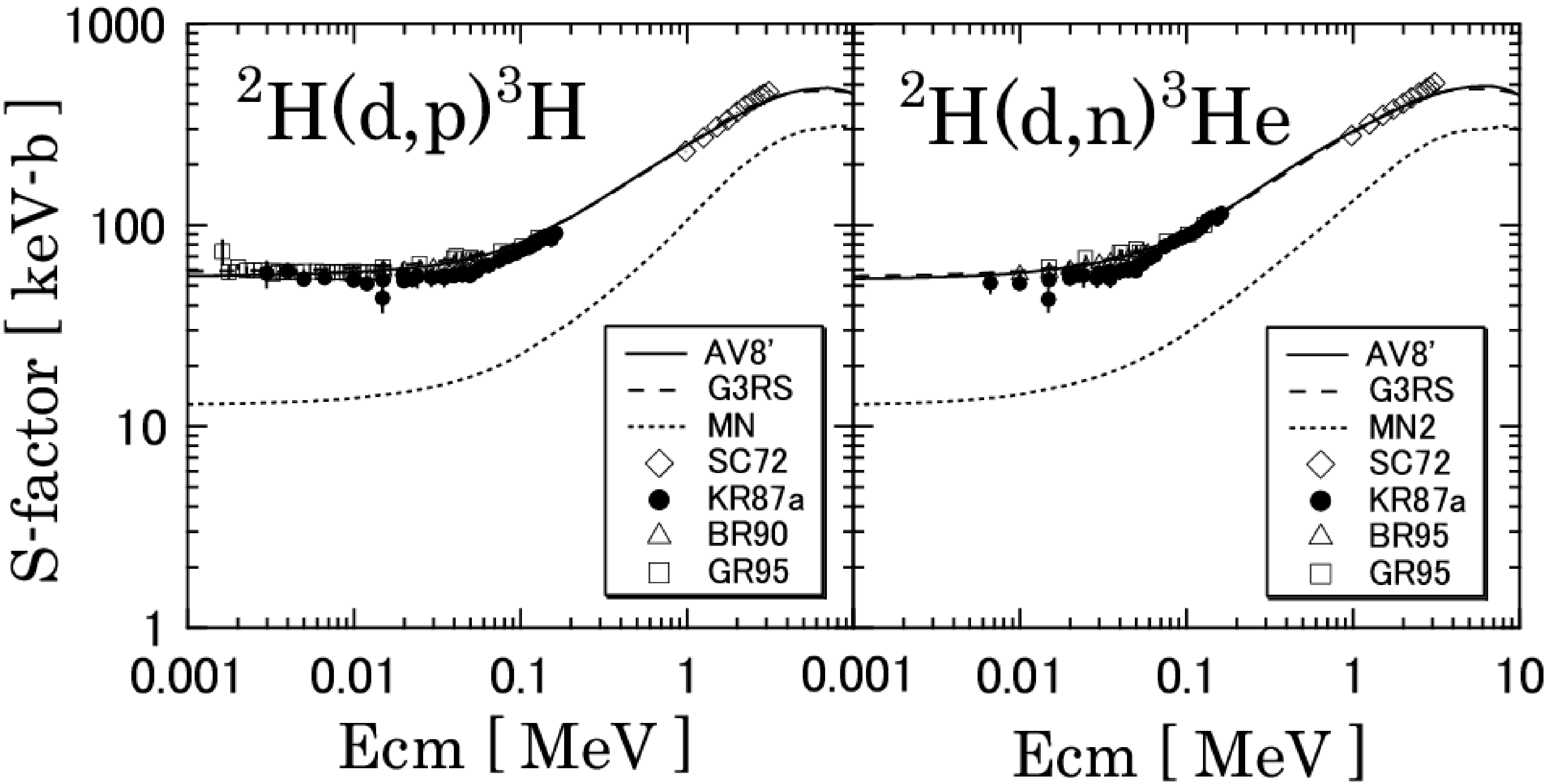
We use 3 NN interactions:

- Realistic: Argonne AV8', G3RS
- Effective: Minnesota MN



- No parameter
- MN does not reproduce the plateau (no tensor force)
- D wave component in ^{4}He:
  13.8% (AV8')
  11.2% (G3RS)

Transfer reactions $^2H(d,p)^3H$, $^2H(d,n)^3He$



- MN force strongly understimates the data
- Good agreement with realistic force → importance of the tensor force

Needs for nuclear astrophysics:

- low energy cross sections
- resonance parameters

Experiment: direct and indirect approaches

Theory: various techniques

- **fitting procedures** (R matrix)→ extrapolation
- **microscopic models**:
  - ➢ cluster: developed since 1960's, applied to NA since 1980's
  - ➢ ab initio: problems with scattering states, resonances→ limited at the moment
- non-microscopic models: potential, DWBA, etc.
- Current challenges: new data on $^{3}He(\alpha,\gamma)^{7}Be$, triple α process, $^{12}C(\alpha,\gamma)^{16}O$, etc.
  heavier nuclei